Specification

Future Circular Collider Study

Lepton Collider Parameters

WBS PATH
1.4.1.2

ABSTRACT:
The goal of the FCC-ee Lepton Collider is to provide $e^+e^-$ collisions in the beam energy range of 40 to 175 GeV. The main centre-of-mass operating points with large physics interest are 91 GeV (Z-pole), 160 GeV (W pair production threshold), 240 GeV (Higgs resonance) and 350 GeV (ttbar threshold). The expected machine circumference ranges between 80 and 100 km. The machine should accommodate four experiments operated simultaneously and deliver peak luminosities above $1 \times 10^{34} \text{cm}^{-2}\text{s}^{-1}$ per experiment at the ttbar threshold and higher luminosities at lower energies. This document summarizes the baseline parameters for this collider.

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1. Introduction

1.1 Purpose

The purpose of this document is to define a parameter baseline for the lepton collider (FCC-ee) of the Future Circular Collider (FCC) study.

1.2 Scope

The goal of the Future Circular Collider lepton collider is to provide $e^+e^-$ collisions in the beam energy range of 40 to 175 GeV [1]. The main centre-of-mass operating points with large physics interest are around 91 GeV (Z-pole), 160 GeV (W pair production threshold), 240 GeV (Higgs resonance) and 350 GeV (ttbar threshold). The machine would have a circumference of the order of 80 to 100 km, allowing to reach these four operating regions with acceptable synchrotron radiation power. A larger machine also reduces the required total RF voltage. The machine should be able to support four experiments operated simultaneously and deliver peak luminosities above $1 \times 10^{34} \text{cm}^{-2}\text{s}^{-1}$ to each experiment at the ttbar threshold and much higher luminosities at lower energies.

1.3 Definitions, Acronyms and Abbreviations

SI units and formatting according to standard ISO 80000-1 on quantities and units are used throughout this document where applicable.

The four baseline operation energies are referred to a Z (91 GeV), W (160 GeV), H (240 GeV) and ttbar (350 GeV) operation points.

- **c.m.** Centre of Mass
- **FCC** Future Circular Collider
- **FODO** Focusing and defocusing quadrupole lenses in alternating order
- **BS** Beamstrahlung
- **IP** Interaction Point
- **FF** Final focus
- **LSS** Long straight section
- **LHC** Large Hadron Collider
- **RF** Radio Frequency
- **SR** Synchrotron Radiation
- **RDP** Resonant Depolarization (for energy calibration)
- **TBC** To Be Confirmed
- **TBD** To Be Defined
1.4 References


[8] A. Bogomyagkov et al., “Beam-Beam Effects Investigation and Parameters Optimization for a Circular e+e- Collider TLEP to study the Higgs Boson”, to be published in PRSTAB.


1.5 Overview

Section 1 gives an overview, explains abbreviations and terms and lists references.

Section 2 introduces assumptions and constraints that impact the parameter set.

Section 3 contains the table of baseline parameters.
2. Collider Parameters

2.1 Baseline layout

The current baseline assumes a layout similar to LEP/LHC with a number of equal-length arcs and long straight sections (LSS). The number of arcs and LSS is currently set to 12. The length of each LSS is 1.5 km and the length of each arc is 6.8 km, leading to a total machine length of 100 km. For a filling factor of 0.84 the corresponding bending radius is 11 km and the required bending field in the dipole magnets ranges from 130 to 540 G.

To accommodate more than 10'000 bunches for operation at the Z pole and many thousand bunches for operation at the W threshold, the two beams must circulate in separate vacuum chambers in the arc sections and in all the LSSs without experiments. This concept is similar to the LHC and to other modern high luminosity factories such as DAFNE, KEKB and PEPII. It is estimated that with a common vacuum chamber for both beams in the entire collider, as was the case for LEP, the number of bunches would be limited to a few hundred.

The straight sections need to accommodate four main experiments. As a baseline it is assumed that the experiments will be distributed equally around the ring as it was the case for LEP [2]. Such a symmetric configuration is generally more favorable to achieve high luminosity and high beam-beam parameters.

Due to the large energy loss per turn at 175 GeV (~4.5%) it is favorable to distribute the RF system evenly over the ring. At this stage it is assumed that the RF is distributed over all 12 LSS. The optimization of the RF system layout is an important item of the FCC-ee study.

The other LSSs will be used for injection, beam dump and collimation systems. The detailed layout will be done in conjunction with the FCC-hh, and is not available at this stage.

Staging scenarios for the machine, concerning for example the RF system, will be evaluated during the study. They will be included in future versions of this document.

2.2 Beam parameters

2.2.1 Introduction

A wide parameter space exists for the beam parameters, which is constrained by different limitations. In the following we describe how the current baseline parameter sets were chosen. Detailed studies will be needed to confirm their validity and to determine optimum parameters.

Most critical for the experiments are of course luminosity (L), beam energy (E) and polarization (P).
2.2.2 Luminosity and bunch parameters

Luminosity can be expressed as a function of the beam current \( I \), the vertical head-on beam-beam parameter \( \xi_y \) and the vertical betatron-function at the collision point \( \beta_y^* \) as

\[
L = \xi_y \frac{I}{e} \frac{\gamma \beta_y^*}{r_e} F H
\]

Here \( r_e \) is the classical electron radius and \( e \) its charge. \( I \) is the total beam current and \( \gamma \) the ratio of energy over rest mass of the electron. The form factor \( F (\leq 1) \) corresponds to the geometric luminosity reduction due to the crossing angle between the two beams. For the current parameter set the crossing angle at the IP is zero and \( F \) equals 1. The form factor \( H \) is the hourglass factor (\( \leq 1 \)) giving the luminosity reduction due to the change of the vertical \( \beta^* \) along the bunch. \( H \) becomes significant when the bunch length is larger than \( \beta^* \). \( H \) ranges between 0.65 and 0.85 for FCC-ee.

The total beam current is defined by the acceptable synchrotron radiation power. At this stage the maximum SR power is set to 50 MW per beam for all energies. Since the energy loss per turn scales with \( E^4 \), the total beam current drops \( \propto \frac{1}{E^4} \). The beam current varies between 1.4 A at 45.5 GeV and 7 mA at 175 GeV. The SR power budget does not take into account losses outside of the arcs, for example losses in dipole magnets or other elements required to bring the two beams together in the LSSs. Such losses will be taken into account when work on the machine layout will be sufficiently advanced.

We assume an upper limit for the total beam-beam tune shift \( \xi_y \) scaled roughly from the observations made at LEP [3]. The scaling of the beam-beam parameters for 4 experiments can be expressed as

\[
\xi_y^{\max} \approx 0.5 \frac{0.4}{r_s}
\]

In this equation \( r_s \) is the longitudinal damping rate in turns. For LEP1 at 45.5 GeV, this expression yields a maximum beam-beam parameter of 0.045. Scaled to FCC-ee the maximum parameter at 45.5 GeV is 0.027 which is in good agreement with recent beam-beam simulations [4]. The maximum beam-beam parameters at the other energy points are then estimated to 0.053 (W), 0.087 (H) and 0.137 (ttbar). These estimates are used as guidelines for the definition of the parameters. It must be noted that crab-waist schemes for the final focus at the IP may enable significantly higher beam-beam parameters [5]. Such schemes will be evaluated as part of the study; a first proposal was already made at a recent TLEP workshop [4].

In the approximation of flat beams and for head-on collisions [6], the beam-beam parameters may be expressed in terms of the beam parameters as

\[
\xi_y = \frac{r_e N \beta_y^*}{2\pi \gamma \sigma_x (\sigma_x + \sigma_y)} \approx \frac{r_e N}{2\pi \gamma \epsilon_x}
\]

and
For a given energy the horizontal beam-beam parameter depends only on bunch population $N$ and emittance but not on the horizontal betatron function $\beta_x^*$. $\xi_y$ can thus be controlled through the emittance by adapting the phase advance (see the section on optics below) or the damping partition numbers (e.g. LEP2) or with wigglers (e.g. LEP1).

At very high luminosity photon radiation at the IP in the field of the counter rotating bunch (‘Beamstrahlung’, BS) becomes a limitation to the performance of the colliders [7] [8]. The strength of BS may be expressed by the average bending radius $\rho_{BS}$ of the particles in the field of the opposing beam [8]:

$$\frac{1}{\rho_{BS}} \approx \frac{N r_x}{\gamma \sigma_y^* \sigma_x^*} \sqrt{2}$$

$\sigma_x^*$ is the horizontal beam size at the IP and $\sigma_y$ is the bunch length. $\rho_{BS}$ can be lowered by increasing the bunch length and using flat beams at the IP ($\sigma_x^* >> \sigma_y^*$). The BS lifetime $\tau_{BS}$ can be expressed as

$$\tau_{BS} \propto \frac{\rho_{BS}^{3/2} \eta^{1/2}}{\sigma_x} \exp(A \eta \rho_{BS})$$

A is a constant and $\eta$ is the energy acceptance of the ring (RF and lattice). A large acceptance of $\sim 2\%$ is required at FCC-ee for the H and ttbar operation points. At those operation points the BS lifetime can easily drop to a few minutes only, and a careful optimization of the beam parameters and optics is required to ensure a sufficient lifetime margin. The process requires careful simulation and tracking studies since the equilibrium bunch lengths and the beam sizes are affected by the beam-beam interaction and by BS; a self-consistent calculation is required for reliable predictions. For the Z and W operating points a momentum acceptance of 1% is sufficient.

A lower limit for the vertical beta-function arises from the final focus and the chromatic correction. A very small vertical $\beta^*$ of 1 mm is set as baseline parameter, compared to 40-50 mm at LEP. The FCC-ee target value is still three times larger than the target value of SuperKEKB [4]. The horizontal $\beta^*$ on the other hand must be kept large (around 0.5-1 m) since flat beams with large $\sigma_x^*$ improve the BS lifetime.

The luminosity lifetime is limited by the burn-off of the beam which is dominated for FCC-ee by radiative Bhabha scattering. At LEP the effective total cross-section for radiative Bhabha scattering was observed to be approximately independent of energy, the absence of the expected energy dependence could be explained by field screening inside the bunch. A cross-section of $\sigma_{ee} \approx 0.21$ (b) was measured at LEP [2], in good agreement with simulations with the program BBREM[2]. The program BBBREM was used to estimate the cross-section for FCC-ee for a momentum acceptance of 2%. The energy dependence was found to be very weak, in agreement with the LEP observations. A central value of $\sigma_{ee} = 0.15$ (b) was choosen for FCC-ee. A more accurate value for the
cross-section will be evaluated in the future for the final energy acceptance. The beam lifetime due to radiative Bhabha scattering can easily be calculated as

\[ \tau_L = \frac{I}{e L n_{IP} \sigma_{ee}} \]

The lifetime depends of course directly on the number of IPs \( n_{IP} \). At the H and ttbar operation points \( \tau_L \) can be as low as \( \sim20 \) minutes.

Other limitations for beam parameters need to be studied, e.g. collective instabilities and electron cloud effects. These may restrict the range of available choices.

The current baseline assumes head-on or nearly head-on collisions with small crossing angles (0.1 - 1 mrad). A crab-waist scheme has recently been proposed for TLEP [8]; it could be well adapted to the large number of bunches required for the Z and W operating points. Such a scheme will be studied in more detail, in particular the integration with the experiment, and it may become part of a future baseline.

2.3 Accelerator lattice

The arc lattice design work has just started using a FODO cell with a length of 50 m as basis for the ttbar operating point. At the lower energies the phase advance will be reduced to maintain a sufficiently large emittance (control of the horizontal beam-beam parameter). In the parameter table given at the end, it is currently assumed that the phase advance is lowered by increasing the length of the cell in units of the basis cell. For the W working point the cell length is doubled, for the Z working point it is increased by a factor of 6. Within the parameter table the momentum compaction factor and the horizontal emittance were scaled according to the following scaling laws

\[ e_x \propto \frac{E^2}{J_x Q^3} \]

and

\[ \alpha_x \propto \frac{1}{Q} \]

Q is the horizontal tune, with \( Q(ttbar) = Q(H) = 2 \times Q(W) = 6 \times Q(Z) \). The total tune is given roughly by \( Q(ttbar) \sim400 \), and \( \alpha_x(t) \sim5E-6 \). \( J_x \) is the horizontal damping partition number, \( J_x=1 \) for the current FCC-ee parameter set.

The design of a final focus with vertical \( \beta^* \) of 1 mm with an energy acceptance of around 2% to control the BS lifetime at the H and ttbar operation points has started. The chromaticity must be corrected locally with sextupoles inserted in the FF.

2.4 RF parameters

For the present baseline an 800 MHz RF system is considered as a default to maintain sufficiently short bunch lengths. The total RF voltage at the ttbar threshold must be in the range of 11 to 12 GV depending on the final choice of the circumference. It is
currently assumed that the effective length of the RF system will be 600 m for a maximum gradient of 20 MV/m for a given beam.

The option of using a 400 MHz or lower frequency RF system will be part of the study. For an RF frequency of 400 MHz the achievable luminosity is reduced by around 10 - 15% as compared to the baseline table at the end of the document. This moderate reduction is due to the fact that BS at the IP gives a large contribution to the equilibrium energy spread and bunch length.

An important optimization concerns the total size of the RF system. Due to lower RF voltage requirements, less than half of the full RF cavity system is needed for the Z, W and H operation points. In combination with the very high beam currents it may be possible to split the full RF system in two parts that are dedicated to a single beam (300 m per beam) at those energies. At the highest energy (ttbar) the same RF system may be used for both beams as was the case for LEP, requiring however a change of the machine layout for ttbar operation. Such considerations require a careful optimization of the ring layout.

2.5 Injector chain and booster ring

The short lifetimes from radiative Bhabha scattering and from BS, potentially as low as 10 minutes or less, require continuous top-up injection. The FCC-ee collider will therefore be operated at constant energy and must be continuously filled by a cycling booster ring. The booster is installed in the FCC tunnel and has its own RF system, with same length and total voltage as for the FCC-ee collider. The booster ring will be used alternatively for e^+ and e^- beams. The currents in the booster will however be much lower than in the collider ring, at the level of few percent of the full current of the Z point, which corresponds to a few tens of mA. The required RF power is therefore much lower (more than a factor 10). The booster will be cycling between its injection energy of 10 - 40 GeV and the operating energy of FCC-ee. The lower limit for injection energy may be given by magnetic field, aperture and impedance considerations. The higher limit on the energy will be defined by the cost and complexity of the injector chain. The maximum repetition rate is currently estimated to be around 0.1 Hz. The required flux of e^+ and of e^- is currently estimated to be 2x10^{12} particles per second for each species. This number does not take into account transfer efficiencies between the different elements of the accelerator chain.

2.6 Polarization

Beam polarization is an important parameter for operation at the Z and W.

Resonant depolarization (RDP) of a transversely polarized beam provides an exceptionally accurate measurement of the beam energy, to the level of 0.1 MeV or better. Such accuracy is required to reduce by an order of magnitude the uncertainties on the Z boson mass and width [1]. Such an improvement of the accuracy requires continuous monitoring of the beam energy with for example polarized non-colliding bunches. With the very small momentum compaction factor of FCC-ee (at level of few
1E-5), the range of energy variations over a year is expected to be larger than 100 MeV (earth tides, geological movements etc). The interpolation of the average beam energy as determined by RDP to the IPs requires an excellent understanding of all sources of energy loss and energy gain (for example RF cavity voltages and phases).

Electron (and positron) beams polarize spontaneously in storage rings due to the emission of synchrotron radiation up to an equilibrium level of 92.4%. The build-up time of polarization $\tau_p$ scales like

$$\tau_p \propto \frac{D^3}{E^5}$$

Compared to LEP1, $\tau_p$ is increased by a factor $\sim 4^3$ to around 200 hours which is clearly excessively long. The rise time may be lowered to $\sim 12$ hours using wigglers at the price of a lower beam current to compensate the large (ten's of MW) power loss from the wigglers [4]. Such a rise time may be acceptable for energy calibration that only requires a few percent of polarization. Besides the issue of power, the use of wigglers is also limited by the induced energy spread. The LEP observations [1][2] indicate that the maximum tolerable energy spread is $\sim 70$ MeV (compared to the 440 MeV spacing of the integer spin resonances). For such a limit spontaneous polarization should be observable without wigglers at the W operation point of FCC-ee with $\tau_p \sim 10$ hours, as compared to LEP, where the larger energy spread prevented build-up of polarization at the W threshold.

Longitudinal polarization at the IP can be used for precise Left-Right asymmetry measurements at the Z pole. Such an option becomes interesting for polarization levels of 30% or more. Spin rotators would have to be installed around each IP where data taking with longitudinal polarization is foreseen.

Reaching at the same time a high level of polarization and a reasonable polarization time is a considerable challenge, requiring cancellation of depolarizing effects at a level of perfection that is much better than achieved in LEP. Various ideas will be investigated, such as Siberian snakes in the storage ring itself, or injection of a polarized beam from the booster or from a dedicated polarizing damping ring in combination with Siberian snakes. Such options must be studied in detail.

2.7 Further studies

The presented parameter list is tentative and meant to provide a basis for further studies. More work is required to fully establish the feasibility of these parameters, in particular:

- The achievable beam-beam parameters.
- The beamstrahlung lifetime.
- The achievable $\beta_y^*$. 


• The layout of the interaction regions and the separation-recombination schemes for the two beams.

• Means to obtain highly polarized beams.

Once the feasibility of the parameters has been established further studies will be required to optimise them.

Careful optimization of the layout and of the designs of absorbers for synchrotron radiation will be required.
### 3. Parameter Overview

The baseline parameters of FCC-ee are compared to LEP1 and LEP2 parameters in the following table.

The LEP parameters only represent a snapshot of the LEP parameter space which includes many more energy points and different modes of operation (Pretzel, bunch train, large Jₓ, etc) [2]. The LEP1 configuration given in the table corresponds to Pretzel operation in 1994 where the highest beam-beam parameters were achieved. 50% higher luminosities were obtained with bunch trains, but with lower beam-beam parameters. The LEP2 configuration corresponds roughly to the maximum energy (but without “special tricks” as increase of Jₓ and spreading of the bending field on orbit correctors).

The energy acceptance η of FCC-ee is assumed to be 2% at all energies, but it should be noted that such a large acceptance is not required at the Z and W operation points.

The emittance ratio is set to 0.1% or larger, with the constraint that the absolute vertical emittance should not be smaller than ~2 pm; this is already a very small value for such a large machine.

Both energy spread and bunch length are quoted for a non-colliding beam, where the values are given by the equilibrium spread from synchrotron radiation in the arc, and for colliding beams which includes photon emission at the IP (BS). As can be seen in the table there is a significant blow-up of the beams due to radiation at the IP. These results are consistent with independent simulation presented at the TLEP workshop [4].

The FCC-ee beam-beam parameter values for Z and W are slightly higher than the estimated value from the LEP data (see Section 2.2.2); they are however consistent with tracking results [4].

The beam-beam parameter and the luminosity at the ttbar threshold can possibly be increased in the future if emittance ratios below 1/1000 and vertical emittances at the level of 1 pm or less are achievable. The current parameter set corresponds to a BS lifetime of around 20 minutes.

For a 400 MHz RF system the predicted luminosities decrease by 10-15% if no other parameter is changed. The change in bunch length may however provide room for re-optimization of the parameters, in particular with respect to BS, in particular at the ttbar threshold.
Table 1: FCC-ee baseline parameters compared to LEP1 and LEP2.

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<td>0.09</td>
<td>0.14</td>
<td>0.19</td>
</tr>
<tr>
<td>- Total (including BS)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bunch length [mm]</td>
<td>8.6</td>
<td>11.5</td>
<td>1.64</td>
<td>1.01</td>
<td>0.81</td>
<td>1.16</td>
</tr>
<tr>
<td>- Synchrotron radiation</td>
<td>8.6</td>
<td>11.5</td>
<td>2.56</td>
<td>1.49</td>
<td>1.17</td>
<td>1.49</td>
</tr>
<tr>
<td>- Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy loss / turn [GeV]</td>
<td>0.12(1)</td>
<td>3.34</td>
<td>0.03</td>
<td>0.33</td>
<td>1.67</td>
<td>7.55</td>
</tr>
<tr>
<td>SR power / beam [MW]</td>
<td>0.3(1)</td>
<td>11</td>
<td>4</td>
<td>5.5</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Total RF voltage [GV]</td>
<td>0.24</td>
<td>3.5</td>
<td>2.5</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>RF frequency [MHz]</td>
<td>352</td>
<td>800</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal damping time (\tau_E) [turns]</td>
<td>371</td>
<td>31</td>
<td>1320</td>
<td>243</td>
<td>72</td>
<td>23</td>
</tr>
<tr>
<td>Energy acceptance RF [%]</td>
<td>1.7</td>
<td>0.8</td>
<td>2.7</td>
<td>7.2</td>
<td>11.2</td>
<td>7.1</td>
</tr>
<tr>
<td>Synchrotron tune (Q_s)</td>
<td>0.065</td>
<td>0.083</td>
<td>0.65</td>
<td>0.21</td>
<td>0.096</td>
<td>0.10</td>
</tr>
<tr>
<td>Polarization time (\tau_P) [min]</td>
<td>252</td>
<td>4</td>
<td>11200</td>
<td>672</td>
<td>89</td>
<td>13</td>
</tr>
<tr>
<td>Hourglass factor H</td>
<td>1</td>
<td>1</td>
<td>0.64</td>
<td>0.77</td>
<td>0.83</td>
<td>0.78</td>
</tr>
<tr>
<td>Luminosity/IP (10^{34} \text{cm}^{-2}\text{s}^{-1})</td>
<td>0.002</td>
<td>0.012</td>
<td>28.0</td>
<td>12.0</td>
<td>6.0</td>
<td>1.8</td>
</tr>
<tr>
<td>Beam-beam parameter</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Horizontal</td>
<td>0.044</td>
<td>0.040</td>
<td>0.031</td>
<td>0.060</td>
<td>0.093</td>
<td>0.092</td>
</tr>
<tr>
<td>- Vertical</td>
<td>0.044</td>
<td>0.060</td>
<td>0.030</td>
<td>0.059</td>
<td>0.093</td>
<td>0.092</td>
</tr>
<tr>
<td>Luminosity lifetime <a href="2">min</a></td>
<td>1750</td>
<td>434</td>
<td>298</td>
<td>73</td>
<td>29</td>
<td>21</td>
</tr>
<tr>
<td>Beamstrahlung critical</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
</tbody>
</table>

(1) Does not take into account the contribution of damping and emittance wigglers.
(2) The luminosity lifetime corresponds to 4 IPs.
4. Appendix

4.1 Energy spread and bunch length with beamstrahlung

The strength of the beamstrahlung is characterized by a parameter \( \gamma \) that can be expressed as \([10][11]\)

\[
\gamma \approx \frac{5}{6} \frac{r_c^2 \gamma N}{\alpha \sigma_r (\sigma_r^+ + \sigma_r^-)}
\]

Here \( \alpha \) represents the fine structure constant, \( \alpha \approx 1/137 \). The average number of photons emitted per collision is roughly given by

\[
n_\gamma \approx 2.54 \frac{\alpha \sigma_r}{\bar{\lambda}_e \gamma} \left( \frac{\gamma}{1 + \gamma^2/3} \right)^{1/2}
\]

and the average energy loss by

\[
\delta_B \approx 1.24 \frac{\alpha \sigma_r}{\bar{\lambda}_e \gamma} \left( \frac{\gamma^2}{1 + (1.5 \gamma)^2} \right)
\]

\( \bar{\lambda}_e \) is the electron Compton wavelength divided by \( 2\pi, \bar{\lambda}_e = r_c/\alpha \). The standard deviation of the energy loss is given by:

\[
\sigma_{\delta,B} \approx \delta_B \left[ 0.333 + \frac{4.583}{n_\gamma} \right]^{1/2}
\]

The additional energy spread due to beamstrahlung (to be added in quadrature) can now be estimated from

\[
\Delta \sigma_{\delta,B} \approx \frac{1}{2} \sqrt{\frac{r_e n_{\text{ip}}}{T_0} \sigma_{\delta,B}}
\]

Leading to a total relative energy spread of

\[
\sigma_\delta = \sqrt{\Delta \sigma_{\delta,B}^2 + \sigma_{\delta,SR}^2}
\]

Using \( \sigma_\delta = \sigma_s \alpha_{\delta,SR}/\sigma_{\delta,SR} \), a self-consistent analytical solution for \( \sigma_\delta \) follows from

\[
\sigma_\delta^2 - \sigma_{\delta,SR}^2 = \left( \frac{\sigma_{\delta,SR}}{\sigma_\delta} \Delta \sigma_{\delta,B} \right)^2
\]

\[
\sigma_{\delta,\text{tot}} = \sigma_{\delta,\text{tot}} \frac{\sigma_{\delta,SR}}{\sigma_{\delta,SR}}
\]

The self-consistent full energy spread \( \sigma_\delta \) can be obtained iteratively from the equations above, starting from \( \sigma_{\delta,SR} \) and \( \sigma_{\delta,SR} \) and updating \( \sigma_\delta \) and \( \sigma_s \) at each step until the values converge.
4.2 Radiative Bhabha cross-section

The radiative Bhabha cross-section was estimated using the program BBBREM [9]. Its dependence on the momentum aperture of the accelerator is shown in Figure 1. The energy dependence of the cross-section for a momentum aperture of 2% is shown in Figure 2.

![Figure 1: Evolution of the radiative Bhabha cross-section evaluated with BBBREM as a function of the momentum aperture at a beam energy of 120 GeV.](image1)

![Figure 2: Dependence of the radiative Bhabha cross-section on the beam energy for a momentum acceptance of 2%.](image2)
4.3 Parameter table updates

The table presented in this document was updated in Version 1.0 with respect to the Table (circumference 100 km) used as basis for many presentations at the TLEP workshop [4].

The following changes were applied:

- To reduce the number of configurations of the optics at this stage the horizontal $\beta^*$ at the W was changed from 0.2 to 0.5 m to be identical to the Z and H values. The vertical emittance and bunch currents were adjusted to re-optimize somewhat the luminosity.

- The momentum compaction factor values were increased for all energies by a factor 5 to be consistent with the first available lattice. At 175 GeV and 80 km circumference this lattice has $\alpha_c = 6.6E-6$ which was scaled $\sim 1/R$ to 5E-6 for a 100 km ring. In the TLEP workshop table, $\alpha_c = 1E-6$ at 175 GeV. This change induces an increase in the bunch length which lowers the luminosity by $\sim 5$-10%.

- To compensate the increase in bunch length due to $\alpha_c$, the RF voltage was increased at the Z (2 to 2.5 GV), W (2 to 4 GV). For the ttbar operating point the RF voltage was scaled to 11 GV (from 12 GV), for the H operating point to 5.5 GV (from 6 GV).

- The optics at the H operating point was chosen to be identical to the ttbar point (instead of using the same as for the W). This lowers the horizontal emittance from 7.5 to 0.94 nm. To maintain the same value for the beam-beam parameter, the bunch population is lowered from 3.7 to 0.46E11 particles/bunch, while the number of bunches is increased from 170 to 1350 to maintain a constant SR power. The vertical emittance is lowered from 16 to 1.9 pm. With the smaller momentum compaction factor, the bunch length decreases significantly, while the Hour-glass factor H increases leading to a final luminosity of 6E34 cm$^{-2}$s$^{-1}$.

- The increase of the momentum compaction factor and the associated bunch shortening were used to increase the bunch population at the ttbar threshold from 0.86E11 to 1.4E11 particles per bunch while maintaining a sufficient BS lifetime. The number of bunches was reduced accordingly from 130 to 98. This brings the luminosity close to 2E34 cm$^{-2}$s$^{-1}$.

- The beam-beam parameter at the Z was lowered from 0.078 to 0.03 taking into account LEP observations and scaling laws as well as simulations presented at the last TLEP workshop [4]. At the W the beam-beam parameter was lowered from 0.086 to 0.060 based on the same arguments.

- The radiative Bhabha cross-section value was changed from 0.21 barn (LEP value) to 0.15 barn to account for the large momentum acceptance of 2% that is used as baseline for FCC-ee. The luminosity lifetimes were updated in the baseline parameter table, all values were increased by a factor 0.21/0.15 = 1.4.